OCR
RECOGNISING ACHIEVEMENT

Wednesday 25 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4763 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The surface tension of a liquid enables a metal needle to be at rest on the surface of the liquid. The greatest mass m of a needle of length a which can be supported in this way by a liquid of surface tension S is given by

$$m = \frac{2Sa}{g}$$

where g is the acceleration due to gravity.

(i) Show that the dimensions of surface tension are MT^{-2}. **[3]**

The surface tension of water is 0.073 when expressed in SI units (based on kilograms, metres and seconds).

(ii) Find the surface tension of water when expressed in a system of units based on grams, centimetres and minutes. **[3]**

Liquid will rise up a capillary tube to a height h given by $h = \frac{2S}{\rho g r}$, where ρ is the density of the liquid and r is the radius of the capillary tube.

(iii) Show that the equation $h = \frac{2S}{\rho g r}$ is dimensionally consistent. **[3]**

(iv) Find the radius of a capillary tube in which water will rise to a height of 25 cm. (The density of water is 1000 in SI units.) **[2]**

When liquid is poured onto a horizontal surface, it forms puddles of depth d. You are given that $d = kS^{\alpha}\rho^{\beta}g^{\gamma}$ where k is a dimensionless constant.

(v) Use dimensional analysis to find α, β and γ. **[4]**

Water forms puddles of depth 0.44 cm. Mercury has surface tension 0.487 and density 13 500 in SI units.

(vi) Find the depth of puddles formed by mercury on a horizontal surface. **[3]**

2 A light inextensible string of length 5 m has one end attached to a fixed point A and the other end attached to a particle P of mass 0.72 kg.

At first, P is moving in a vertical circle with centre A and radius 5 m. When P is at the highest point of the circle it has speed $10\,\mathrm{m\,s^{-1}}$.

(i) Find the tension in the string when the speed of P is $15\,\mathrm{m\,s^{-1}}$. **[5]**

The particle P now moves at constant speed in a horizontal circle with radius 1.4 m and centre at the point C which is 4.8 m vertically below A.

(ii) Find the tension in the string. **[3]**

(iii) Find the time taken for P to make one complete revolution. **[4]**

Another light inextensible string, also of length 5 m, now has one end attached to P and the other end attached to the fixed point B which is 9.6 m vertically below A. The particle P then moves with constant speed $7\,\mathrm{m\,s^{-1}}$ in the circle with centre C and radius 1.4 m, as shown in Fig. 2.

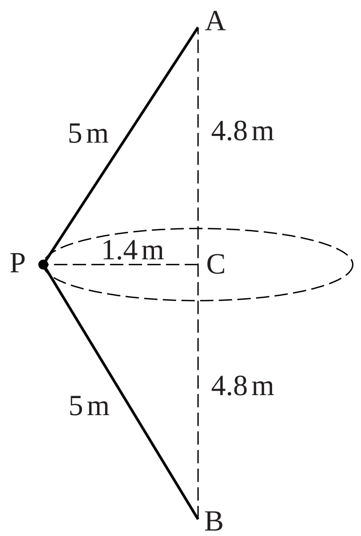

Fig. 2

(iv) Find the tension in the string PA and the tension in the string PB. **[6]**

3 A bungee jumper of mass 75 kg is connected to a fixed point A by a light elastic rope with stiffness $300\,\mathrm{N\,m^{-1}}$. The jumper starts at rest at A and falls vertically. The lowest point reached by the jumper is 40 m vertically below A. Air resistance may be neglected.

(i) Find the natural length of the rope. **[4]**

(ii) Show that, when the rope is stretched and its extension is x metres, $\ddot{x} + 4x = 9.8$. **[3]**

The substitution $y = x - c$, where c is a constant, transforms this equation to $\ddot{y} = -4y$.

(iii) Find c, and state the maximum value of y. **[3]**

(iv) Using standard simple harmonic motion formulae, or otherwise, find

(A) the maximum speed of the jumper,

(B) the maximum deceleration of the jumper. **[3]**

(v) Find the time taken for the jumper to fall from A to the lowest point. **[5]**

4 **(a)** The region T is bounded by the x-axis, the line $y = kx$ for $a \leqslant x \leqslant 3a$, the line $x = a$ and the line $x = 3a$, where k and a are positive constants. A uniform frustum of a cone is formed by rotating T about the x-axis. Find the x-coordinate of the centre of mass of this frustum. **[6]**

(b) A uniform lamina occupies the region (shown in Fig. 4) bounded by the x-axis, the curve $y = 16(1 - x^{-\frac{1}{3}})$ for $1 \leqslant x \leqslant 8$ and the line $x = 8$.

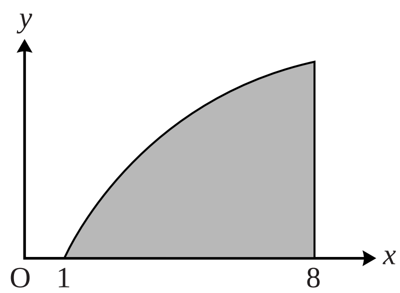

Fig. 4

(i) Find the coordinates of the centre of mass of this lamina. **[8]**

A hole is made in the lamina by cutting out a circular disc of area 5 square units. This causes the centre of mass of the lamina to move to the point (5, 3).

(ii) Find the coordinates of the centre of the hole. **[4]**